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ON THE CORRELATION RADIOMETER TECHNIQUE, II

K. Fujimoto

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Investigation of
Receiver Techniques and Detectors for Use at Millimeter and Submillimeter Wavelengths

Subject of Report
On the Correlation Radiometer Technique, II

Submitted by
K. Fujimoto
Antenna Laboratory
Department of Electrical Engineering

Date
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ON THE CORRELATION RADIOMETER TECHNIQUE, II

I. INTRODUCTION

An analysis of the correlation radiometer technique has been previously reported.[1] The reason for using the correlation technique is essentially to obtain high signal-to-noise ratio as a result of the high correlation between two signals and the low correlation between two noises (cf., Fig. 1). Usually the two signals are considered to be

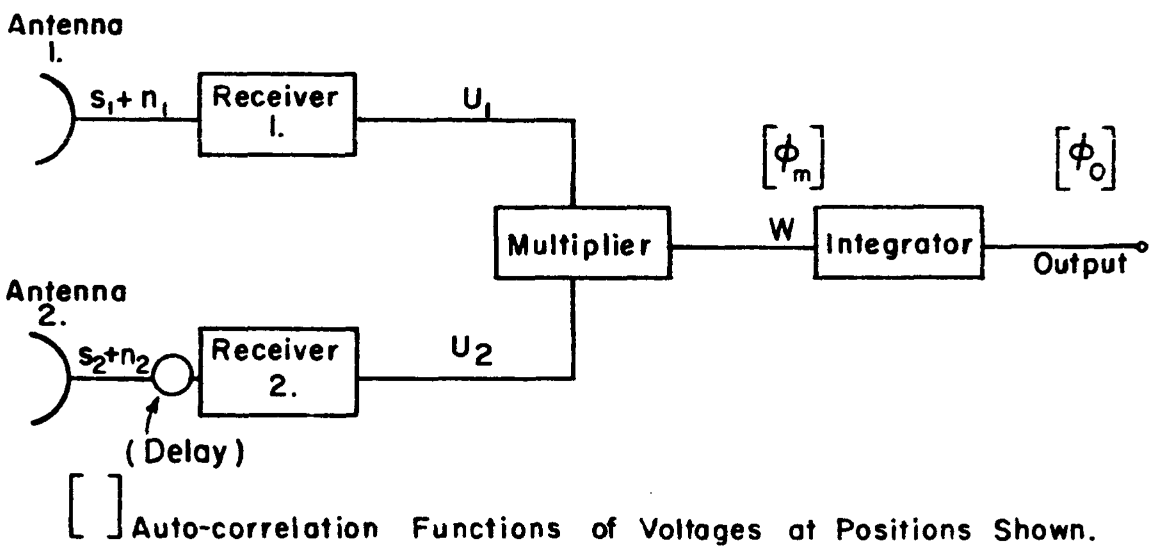

Fig. 1. Simple block diagram for the IF type of correlation radiometer.

coherent in both amplitude and phase over the desired spectrums and during the observation time. However, in practice, one cannot always expect such a perfect coherency between two signals, since one or the other of the signals may be perturbed by the medium through which the signals are passing on their way from the source to the radiometer. In some cases the phase coherency is almost totally destroyed and only the amplitudes of two signals retain any degree of correlation. In order to apply the correlation technique in this case, one must employ square-law envelope detection before the correlation process, as is shown in Fig. 2. We distinguish between the two types of radiometers by calling the first type (where the signals themselves are correlated) the IF type, and the second type (where only the amplitudes are correlated) the ENVELOPE type. (Hereafter we will call these the IF type and ENV type, respectively.)

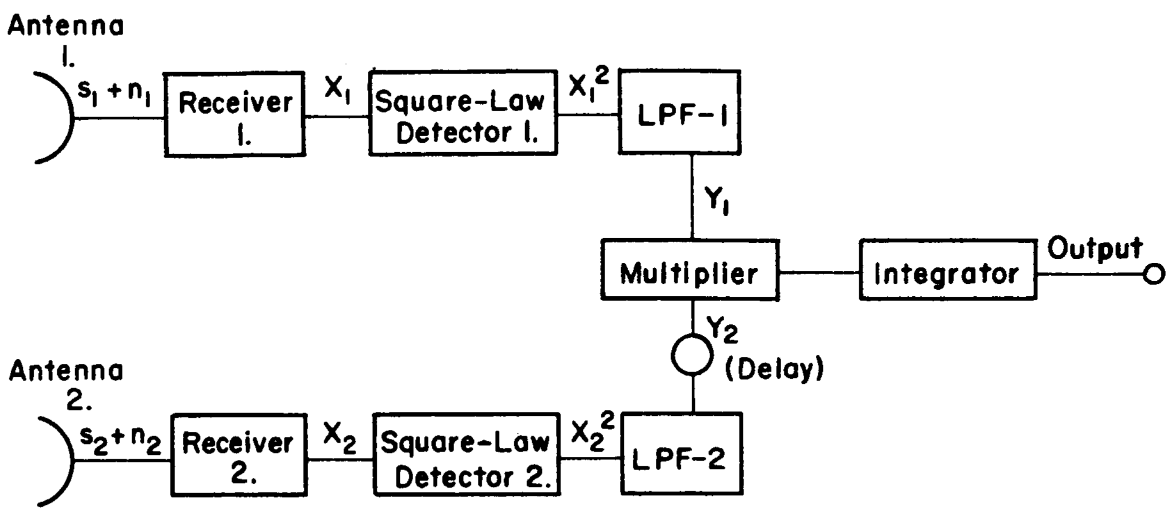

Fig. 2. Simple block diagram for the ENV type of correlation radiometer.

The object of this report is to study the properties of these two types of radiometer separately. (In Reference 1 these two types were not treated separately at all, except as related to the bandwidth of the two types of systems.)

In this report, the expression for signal-to-noise ratio and the minimum detectable temperature will be determined for the ENV type of correlation radiometer, and it will be compared with similar equations for the IF type and the Dicke radiometers which have already been given in Reference 1.

Special features of the correlation radiometer have been discussed in Reference 1. But in view of distinction of two types of system, they are supplementally described as follows:

(1) Disadvantages of the correlation radiometer

(a) Because of the need for two identical receivers, the correlation radiometer systems are necessarily more complex.

(b) Fluctuations in the gain or phase characteristics of the receivers in the correlation radiometer will cause greater output fluctuations than in the case of the Dicke radiometer, where the temperature of the matched load may be made almost the same as the apparent source temperature to reduce the effects of gain fluctuations. Also, phase shifts in the Dicke radiometer receiver have little or no effect upon the output, while in the case of the IF type of correlation system spurious phase shifts in either one of the two receivers will seriously degrade the performance of the radiometer. (Section 2.4 in Reference 1 should be disregarded because of erroneous treatment caused by misuse of uncorrected equations in Reference 2.)

In the ENV type of system, the phase shift of the received signal itself will not introduce any errors in the output, since all of the phase information is lost in the square-law detector. However, any shift in the phase of the signal envelope will cause output errors.

(2) Advantages

(a) The IF type of correlation radiometer is naturally adapted for use in an interferometer system.

(b) The simultaneous observation of two signals is especially suitable for the ENV type.

II. ANALYSIS

(1) The receiver signals

In order to evaluate the minimum detectable signal in terms of the equivalent minimum detectable temperature increment (ΔT), the same procedure as in Reference 1 will be used; i.e., first we evaluate the signal-to-noise ratio (SNR) at the output in terms of the SNR at the input to the radiometer receivers. We have assumed the following:

(a) Both signal and noise possess Gaussian distributions with zero mean values;

(b) Both functions are independent of other variables, and they have the properties of ergodicity and are stationary.

(c) The signals at the receiver inputs are coherent and each has a mean-square value ψ_{s_j}.

(d) The noise signals at the receiver inputs are uncorrelated and each has a mean-square value ψ_{nj}.

(e) Each receiver has high-Q bandpass-type characteristics centered at ω_0.

The subscript j designates channel or receiver number.

(2) Basic relations pertaining to the correlation of two signals

Referring to Fig. 1, the inputs at the correlator are represented by $U_1(t)$ and $U_2(t+\theta)$, where θ is the time delay of channel 1 as compared to channel 2. The output of the multiplier has the correlation function;

(1) $$\phi_m(\tau, \theta) = \overline{W(t, \theta)\, W(t+\tau, \theta)}$$

where

(2) $$W(t, \theta) = U_1(t)\, U_2(t+\theta).$$

The overhead bar denotes a time average. The portion of $\phi_m(\tau, \theta)$, which is independent of τ (i.e., $\phi_m(\theta)]_{dc}$) represents the desired signal power output, and the portion which is dependent upon τ but has no 'dc' components represents the noise power output of the multiplier circuit. The output of the integrator can be found by convolving the integrator input with its impulse response function. Taking the ratio of the 'dc' to the τ-dependent portion of the correlator output then gives the output SNR, which for arbitrary θ is;

(3) $$\left.\frac{S}{N}\right]_{(\tau=0)} = \frac{\phi_m(\theta)]_{dc}}{\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi_m(\tau', \theta)]_{ac}\, |H(\omega)|^2\, e^{-j\omega\tau'}\, d\omega\, d\tau'}$$

where $H(\omega)$ is the transfer function of the integrator circuit. We will obtain the SNR for the maximum mean-square output signals by setting $\theta = 0$. The sensitivity in terms of the minimum detectable equivalent temperature T_{min} or ΔT will be derived later on.

The above has considered the IF type of correlation radiometer.

For the ENV type of system, completely similar relations can be derived, if U_j is replaced by Y_j as shown in Fig. 2.

(3) Signal-to-noise ratio

Referring to Fig. 1, the input signal to the square-law detector (i.e., the receiver output signal) for each channel is given by

$$U_j(t) = A_j(t) \{s_j(t) + n_j(t)\} \tag{4}$$

where $s_j(t)$ is the input signal and $n_j(t)$ is the equivalent input noise. $A_j(t)$ is the gain of receiver, which in the initial analysis will be taken as constant (i.e., $A_j(t) = A_{jo}$). Because of signal coherency, we assume that

$$s_1(t) = s(t) \tag{5a}$$

$$s_2(t) = \eta\, s(t+\theta) \tag{5b}$$

where η is the amplitude factor and θ is the relative time delay of the channel 1 signal as compared to the channel 2 signal. The same form used in Reference 1 for the autocorrelation functions of both signal and noise that have passed through the high-Q bandpass amplifier,

$$\phi_s = \psi_s\, e^{-\omega_s|\tau|} \cos \omega_o\tau \tag{6a}$$

$$\phi_n = \psi_n\, e^{-\omega_n|\tau|} \cos \omega_o\tau \tag{6b}$$

will again be used. Here ω_o is the center frequency of the amplifier, ω_s is the effective half-bandwidth of signal and ω_n is that of the noise. The dc-component of Eq. (1) is found to be

$$\phi_m(\theta)]_{dc} = \eta^2\, \psi_s^2\, e^{-2\omega_s\theta} \cos^2 \omega_o\theta\, . \tag{7}$$

With the assumption that the integrator was an RC-filter with cutoff frequency ω_L, the output noise term was given by[1]

$$(8)\qquad \phi_o(0,\theta)]_{ac} \doteq \frac{\omega_L}{2}\left[\eta^2\psi_s^2\left\{\frac{1}{2\omega_s}+\left(\frac{1-e^{-2\omega_L\theta}}{\omega_L}+\frac{1}{2\omega_s}\right)e^{-2\omega_s\theta}\cos 2\omega_o\theta\right\}\right.$$

$$\left.+\psi_s\psi_{n_2}\left(\frac{1}{\omega_s+\omega_{n_2}}\right)+\eta^2\psi_s\psi_{n_1}\left(\frac{1}{\omega_s+\omega_{n_1}}\right)+\psi_{n_1}\psi_{n_2}\left(\frac{1}{\omega_{n_1}+\omega_{n_2}}\right)\right].$$

Thus the output SNR can be written as (for $\theta = 0$),

$$(9)\qquad \left.\frac{S}{N}\right]_I = \frac{2\eta^2\psi_s^2\left(\frac{\Delta\omega_{IF}}{\omega_L}\right)}{2\eta^2\psi_s^2+\eta^2\psi_s\psi_{n_1}+\psi_s\psi_{n_2}+\psi_{n_1}\psi_{n_2}}$$

where $\omega_s = \omega_{nj} = \frac{\Delta\omega_{IF}}{2}$ has been assumed. (The subscript I refers to IF-type correlators and E to ENV type.)

For convenience, the SNR at the correlator input will be introduced, that is

$$(10)\qquad \left.\frac{S}{N}\right]_{in,1} = R_{in,1} = \left.\frac{\overline{S_1(t)S_1(t+\tau)}}{\overline{N_1(t)N_1(t+\tau)}}\right]_{\tau=0} = \left.\frac{\phi_s(\tau)}{\phi_{n_1}(\tau)}\right]_{\tau=0} = \frac{\psi_s}{\psi_{n_1}} = R_1$$

for channel 1, and

$$(11)\qquad \left.\frac{S}{N}\right]_{in,2} = R_{in,2} = \left.\frac{\overline{S_2(t)S_2(t+\tau)}}{\overline{N_2(t)N_2(t+\tau)}}\right]_{\tau=0} = \left.\frac{\eta^2\phi_s(\tau)}{\phi_{n_2}(\tau)}\right]_{\tau=0} = \frac{\eta^2\psi_s}{\psi_{n_2}} = \eta^2 R_2$$

for channel 2. Using the above, the SNR at the correlator output is expressed by

$$(12a)\qquad \left.\frac{S}{N}\right]_I = R_I = \frac{2(R_{in,1})(R_{in,2})}{(R_{in,1})(R_{in,2})+(1+R_{in,1})(1+R_{in,2})}\cdot\alpha$$

or

$$(12b)\qquad R_I = \frac{2\eta^2 R_1R_2}{\eta^2R_1R_2+(1+R_1)(1+\eta^2R_2)}\cdot\alpha\ .$$

This equation will be used for obtaining the SNR for the ENV type of system.

(b) ENV type of system

In this case, correlation is performed with respect to the envelope of the signal and noise, and different forms for the signal and noise functions will be used. Since both functions are the output of the bandpass-type amplifier, we may express $U_j(t)$ as:

$$U_j(t) = s_j(t) + n_j(t) \tag{13}$$

$$= x_j(t) \cos \omega_o t - y_j(t) \sin \omega_o t.$$

x_j and y_j can be further decomposed into signal and noise terms as follows:

$$x_j(t) = s_j'(t) \cos \Omega_s(t) + n_j'(t) \cos \Omega_n(t) \tag{14a}$$

$$y_j(t) = s_j'(t) \sin \Omega_s(t) + n_j'(t) \sin \Omega_n(t) \tag{14b}$$

where $\Omega(t)$ is the phase factor, which is also considered to be a statistically independent and random function. The output of the square-law device, i.e.; the envelope of combined signal with noise, is:

$$E_j(t) = x_j^2(t) + y_j^2(t) \tag{15a}$$

$$= (s_j'(t))^2 + (n_j'(t))^2 + 2s_j'(t)\, n_j'(t) \cos \{\Omega_{sj}(t) - \Omega_{nj}(t)\}. \tag{15b}$$

The output of the correlator can be found by correlating $E_1(t)$ and $E_2(t+\theta)$, where θ is relative delay time of channel 1 as compared to channel 2. However, from the preceding section we already know the output SNR of the correlator in terms of the input SNR of the correlator. Hence Eq. (15) can be used for obtaining the correlator input SNR, and then combining it with Eq. (12) will give the correlator output SNR.

The mean-square value of the envelope in each channel is given by*

(16a) $$\left.\overline{E_j(t)E_j(t+\tau)}\right]_{\tau=0} = \left.\overline{\{x_j^2(t) + y_j^2(t)\}\{x_j^2(t+\tau) + y_j^2(t+\tau)\}}\right]_{\tau=0}$$

(16b) $$= \{\phi'_{sj}(0) + \phi'_{nj}(0)\}^2 + 2(\phi'_{sj}(\tau) + \phi'_{nj}(\tau))^2]_{\tau=0}$$

where $\phi_{s'j}(\tau)$ is the correlation function of $s'_j(t)$, etc.

The first term in Eq. (16b) which is independent of τ, represents a dc-term that will be eliminated by a blocking circuit before the correlator. Then the correlator input SNR is given by,

(17) $$R'_{in,\,j} = \frac{\psi^2_{s'j}}{2\psi_{s'j}\psi_{n'j} + \psi^2_{n'j}}$$

where

$$\psi_{s'1} = \psi_{s'} \quad \text{and} \quad \psi_{s'2} = \eta^2\psi_{s'}.$$

The SNR at the square-law detector input R_j is found as $\psi_{sj}/\psi_{nj} = \psi_{sj}/\psi_{nj}$. This is derived** by using Eq. (13),

---

*To obtain Eq. (16b) the upper bound of Eq. (15b) has been used, i.e., $\cos\{\Omega_{s'j}(t) - \Omega_{n'j}(t)\} \leq 1$.

**Equation (18b) has been taken as the upper bound of the autocorrelation functions, i.e.,

$$\left.\overline{U_j(t)\,U_j(t+\tau)}\right]_{\tau=0} = \frac{1}{2}(\phi_{xj}(\tau) + \phi_{yj}(\tau))\cos\omega_0\tau]_{\tau=0}$$
$$= \frac{1}{2}(\phi_{s'j}(\tau)\rho_{s'j}(\tau) + \phi_{n'j}(\tau)\,\rho_{n'j}(\tau))]_{\tau=0}$$

where

$$\rho_{s'j}(\tau) = \overline{\cos\{\Omega_{s'j}(t) - \Omega_{s'j}(t+\tau)\}} \leq 1$$

and

$$\rho_{n'j}(\tau) = \overline{\cos\{\Omega_{n'j}(t) - \Omega_{n'j}(t+\tau)} \leq 1.$$

$$\text{(18a)} \quad \left.\overline{U_j(t)\,U_j(t+\tau)}\right]_{\tau=0} = \psi_{sj} + \psi_{nj}$$

$$\text{(18b)} \quad = \frac{1}{2}\left[\psi_{s'j} + \psi_{n'j}\right].$$

Then the SNR at the correlator input is given by

$$\text{(19a)} \quad R'_{in,1} = \frac{R_1^2}{2R_1 + 1} = \frac{(R_{in,1})^2}{2(R_{in,1}) + 1}$$

and

$$\text{(19b)} \quad R'_{in,2} = \frac{\eta^4 R_2^2}{2\eta^2 R_2 + 1} = \frac{(R_{in,2})^2}{2(R_{in,2}) + 1} \quad .$$

This leads to the following expression for SNR at the output of the correlator

$$\text{(20a)} \quad \left.\frac{S}{N}\right]_E = R_E = \frac{2(R_{in,1})^2\,(R_{in,2})^2}{(R_{in,1})^2(R_{in,2})^2 + (R_{in,1} + 1)^2(R_{in,2} + 1)^2} \cdot \beta$$

or

$$\text{(20b)} \quad R_E = \frac{2(R_1)^2\,(\eta^2 R_2)^2}{(R_1)^2(\eta^2 R_2)^2 + (R_1 + 1)^2(\eta^2 R_2 + 1)^2} \cdot \beta$$

where $\beta = \frac{\Delta\omega_d}{\omega_L}$ ($\Delta\omega_d$ is the bandwidth immediately preceding the correlator).

From the preceding, one can see that $R'_{in,j}$ is always smaller than $R_{in,j}$ and at most $R'_{in,j} = 1/2\ R_{in,j}$ ($\eta^2 \le 1$). In other words, from the standpoint of the SNR, placing the square-law detectors before the multiplier causes loss in the radiometer sensitivity, especially in the case where the receiver input SNR is much less than unity. Thus the use of the ENV type of correlation radiometer should be avoided except in those cases where the IF type is not practical (i.e., where there is no phase coherence between the incoming signals to the two receivers). The relation between $R_{in,j}$ and R_j is shown in Fig. 3, and a plot of R_I and R_E vs R_1 for various values of $\eta(\le 1)$ are shown in Figs. 4 and 5, respectively. These plots assume the condition that $\psi_{n1} = \psi_{n2}$ or $R_1 = R_2$. The solid lines in Figs. 3, 4, and 5 show the unity locus where the input and the output SNR are equal. In the ENV type of system

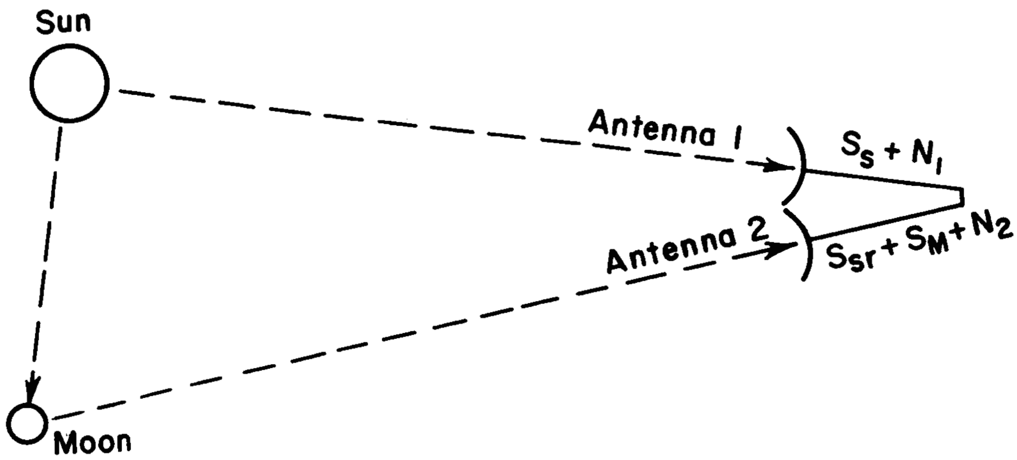

Fig. 3. A proposed experiment to study the bi-static reflection from the moon using a correlation radiometer.

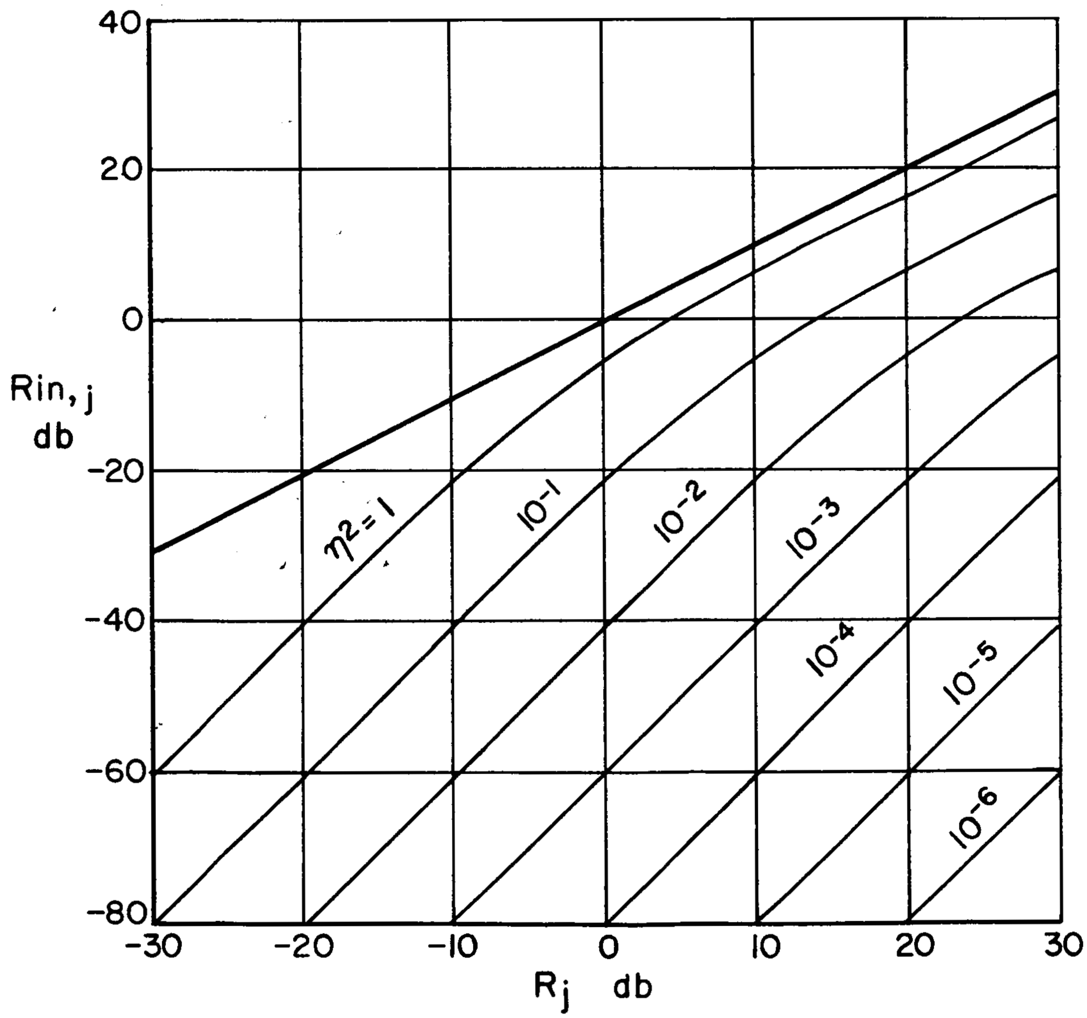

Fig. 4. The correlator input SNR as a function of the receiver input SNR (for j = 1, η^2 = 1).

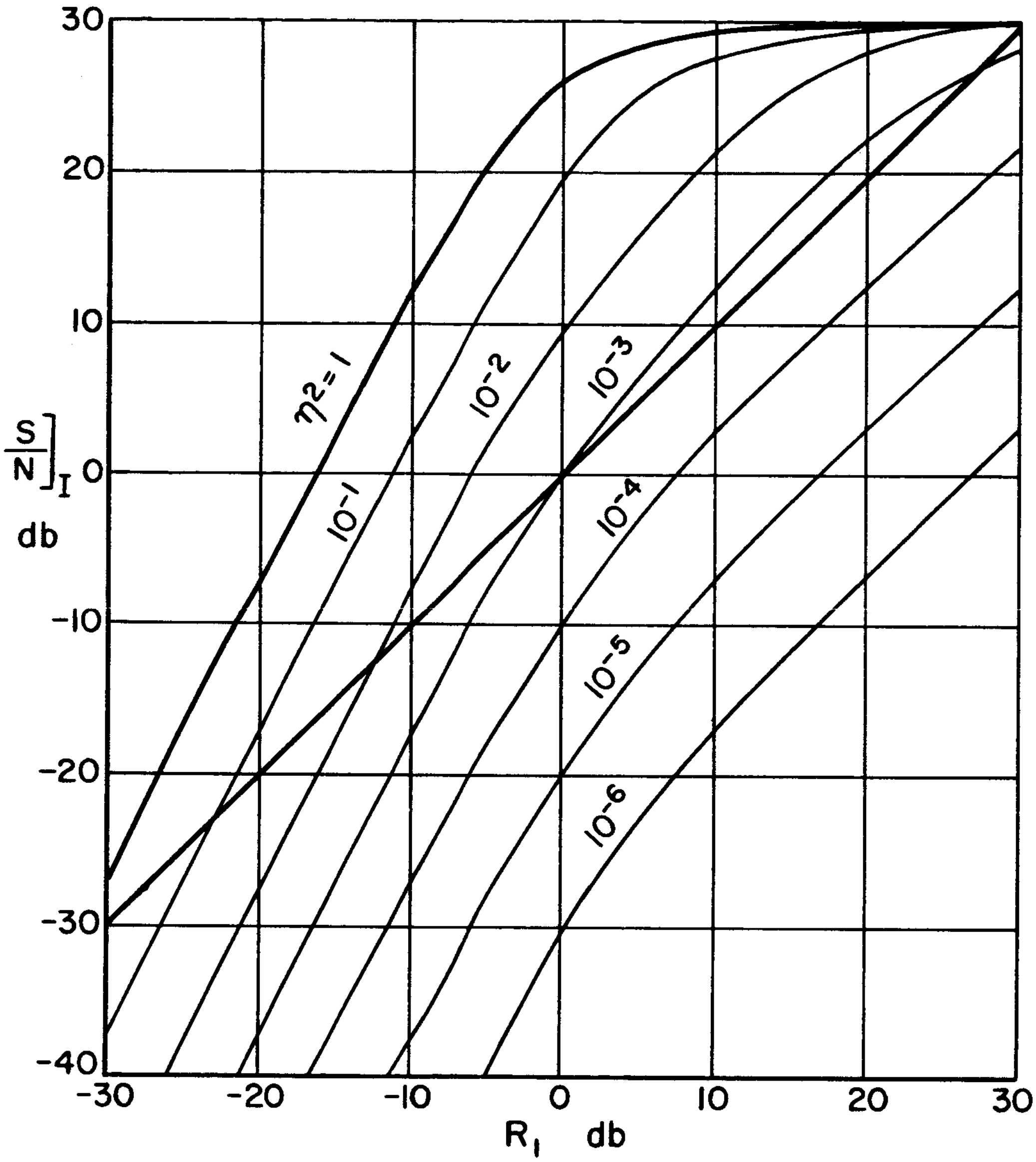

Fig. 5. The output SNR for the IF type of correlation radiometer as a function of the receiver input SNR ($\alpha = 10^3$).

the degradation of the output SNR occurs more rapidly as the input SNR becomes smaller than it does in the IF type of system. However, in either case, since α and β are much greater than unity, we find that one can obtain greater output SNR than input SNR.

(4) The minimum detectable temperature increment

(a) The effect of the input signal strength

We defined ΔT as the minumum incremental variation in the temperature of the source which can be detected, and can replace T_{min} as the measure of sensitivity for the large-signal case.

For the ENV type of system, output signal of the radiometer is given by

$$S = C_E T_1^2 T_2^2 \tag{21}$$

where C_E is a constant. Then we have

$$\Delta T_1 = \frac{1}{4} \cdot T_1 \frac{N}{S} . \tag{22}$$

This ΔT_1 is understood to be equal to ΔT.

Applying this definition, the minimum detectable temperatures for various cases of large and small input SNR can be found and are given in Table 1. As a comparison, Table 2 shows those for the IF type of system.

(5) The effect of the variation of receiver parameters upon the radiometer sensitivity:

(a) Gain fluctuation

Let the gain of the receiver be given by

$$A_j(t) = A_{oj} + \Delta A_j(t) \tag{23}$$

where $\Delta A_j(t)$ is the small variation in the receiver gain. Then the input to the correlator becomes

TABLE 1
(IF Type)

	SNR_{input}	SNR_{output}	T_{min} or	ΔT	Remarks
1	$R_1 >> 1$ $\eta^2 R_2 >> 1$	α		$\frac{T_1}{2\alpha}$	$\left.\frac{S}{N}\right]_I >> 1$
2	$R_1 >> 1$ $\eta^2 R_2 \doteq 1$	$\frac{2}{3}\alpha$		$\frac{3}{4}\frac{T_1}{\alpha}$	$\left.\frac{S}{N}\right]_I >> 1$
3	$R_1 >> 1$ $\eta^2 R_2 << 1$	$2\eta^2 R_2 \alpha$	$\frac{F_e T_o}{2\alpha}$		$\left.\frac{S}{N}\right]_I \leq 1$
				$\frac{T_n}{4\alpha\eta^2}$	$\left.\frac{S}{N}\right]_I >> 1$
	$R_1 \doteq 1$ $\eta^2 R_2 \dot{<<} 1$	$\eta^2 R_2 \alpha$	$\frac{F_e T_o}{\alpha}$		$\left.\frac{S}{N}\right]_I \leq 1$
				$\frac{T_n}{2\alpha\eta^2}$	$\left.\frac{S}{N}\right]_I >> 1$
4	$R_1 << 1$ $\eta^2 R_2 << 1$	$2\eta^2 R_1 R_2 \alpha$	(1) $\frac{F_e T_o}{\sqrt{2\alpha}}$ $T_{n1} = T_{n2}$		$\left.\frac{S}{N}\right]_I \leq 1$
			(2) $\frac{T_o\sqrt{F_{e1} F_{e2}}}{\sqrt{2\alpha}}$ $T_{n1} \neq T_{n2}$		$\eta^2 R_2 = R_1$

Notes: (1) $T_{n1} = T_{n2} = T_n$ and $F_{e1} \doteq F_{e2} = F_e$ have been assumed, otherwise indicated
(2) $T_{nj} = F_{ej} T_o$

TABLE 2
(ENV Type)

	SNR_{input}	SNR_{output}	T_{min} or	ΔT	Remarks
1	$R_1 >> 1$ $\eta^2 R_2 >> 1$	β		$\frac{T_1}{4\beta}$	$\left.\frac{S}{N}\right]_E >> 1$
2	$R_1 >> 1$ $\eta^2 R_2 \doteq 1$	$\frac{2}{5}\beta$		$\frac{5}{8}\ \frac{T_1}{\beta}$	$\left.\frac{S}{N}\right]_E >> 1$
3	$R_1 >> 1$ $\eta^2 R_2 << 1$	$2\eta^4 R_2^2 \beta$	$\frac{F_e T_o}{\sqrt{2\beta}}$		$\left.\frac{S}{N}\right]_E \leq 1$
				$\frac{T_n^2}{8\beta\eta^4}$	$\left.\frac{S}{N}\right]_E >> 1$
	$R_1 \doteq 1$ $\eta^2 R_2 << 1$	$\frac{1}{2}\eta^4 R_2^2 \beta$	$\sqrt{2}\cdot\frac{F_e T_o}{\sqrt{\beta}}$		$\left.\frac{S}{N}\right]_E \leq 1$
				$\frac{T_n^2}{2\beta\eta^4}$	$\left.\frac{S}{N}\right]_E >> 1$
4	$R_1 << 1$ $\eta^2 R_2 << 1$	$2\eta^4 R_1^2 R_2^2 \beta$	(1) $\frac{F_e T_o}{(2\beta)^{\frac{1}{4}}}$ $T_{n1} = T_{n2}$ (2) $\frac{T_o\sqrt{F_{e1} F_{e2}}}{(2\beta)^{\frac{1}{4}}}$ $T_{n1} \neq T_{n2}$		$\left.\frac{S}{N}\right]_E \leq 1$ $\eta^2 R_2 = R_1$

Note: (1) $T_{n1} = T_{n2} = T_n$ and $F_{e1} = F_{e2} = F_e$ have been assumed, otherwise indicated

(2) $T_{nj} = F_{ej} T_o$

(28a) $$\left.\frac{S}{N}\right]_{fE} = \frac{2(R_{in,1})^2(R_{in,2})^2}{(R_{in,1})^2(R_{in,2})^2 + (R_{in,1}+1)^2(R_{in,2}+1)^2(1+\Gamma_E)} \cdot \beta$$

where

(28b) $$\Gamma_E \doteq b_1 + b_2 + b_1 b_2 \ .$$

Let us consider the error in T_{min} due to the gain fluctuation. For a weak signal, one may put S/N = 1. Then $R_{in,1}$ and $R_{in,2}$ are both much less than unity, and we find that

(29) $$\left.T_{min}\right]_{fI} = (T_n/\sqrt{2\alpha})\left(1 + \frac{1}{2}\cdot\Gamma_I\right)$$

for the IF-type system, and

(30) $$\left.T_{min}\right]_{fE} = \left\{T_n/(2\beta)^{\frac{1}{4}}\right\}\left(1 + \frac{1}{4}\cdot\Gamma_E\right)$$

for the ENV-type system. If the same receivers are used in both types of systems, the ENV type would have somewhat larger apparent variations in T_{min} due to gain variations than would the IF type of system, since $\Gamma_E \doteq 8\,\Gamma_I$ and $(2\beta)^{-\frac{1}{4}} > (2\alpha)^{-\frac{1}{2}}$.

For the case of a large signal, i.e., $R_{in,1}$, $R_{in,2} \gg 1$, $\Delta T'$ for each type of system is given by

(31) $$\Delta T'_{fI} = \Delta T_I + \frac{T_1}{4\alpha}\,\Gamma_I$$

for IF type, and

(32) $$\Delta T'_{fE} = \Delta T_E + \frac{T_1}{8\beta}\,\Gamma_E$$

for ENV type.

From the above it may be seen that in the weak-signal case, gain fluctuations are not very important, but in the large-signal case, a variation in the output which is proportional to the input signal is observed. Thus gain fluctuations become very important in the correlation radiometer, since unlike the Dicke radiometer there is no matched load which may be adjusted in temperature to minimize the effects of the receiver gain variations.

(b) the effect of receiver phase-shift variations

As explained in the introduction, phase fluctuations in the receivers of the IF type of system would introduce errors into the output signal, since any variations in the phase characteristics in either one or the other of the receivers would cause at least some incoherency between the two receiver outputs. This degradation of the coherence may be treated as a noise source. In the ENV system, although shifts in the received signal itself will not introduce any errors in the output, any shift in the phase of the signal envelope will cause output errors.

Consider that a phase or time-delay fluctuation, x_θ, is introduced into channel 2 just prior to the correlator. Then

(33a) $$U_1 = U_1(t)$$

(33b) $$U_2 = U_2(t + \theta_r + x_\theta)$$

where θ has been put equal to $\theta_r + x_\theta$. θ_r is the mean value of the channel 2 phase shift and x_θ is the variation in the phase shift. If it can be assumed that x_θ has a Gaussian distribution, then the autocorrelation function of the correlator output is expressed as

(34a) $$\phi_m(\tau,\theta) = \overline{\langle U_1(t)U_2(t+\theta+x_\theta)\, U_1(t+\tau)U_2(t+\theta+x_\theta+\tau)\rangle}_{x_\theta}$$

(34b) $$= \int \overline{U_1\, U_2\, U_{1\tau}\, U_{2\tau}}\; P(x_\theta)\, dx_\theta$$

where $\langle\ \rangle$ denotes the ensemble average over x_θ, and $P(x_\theta)$ is the probability distribution of x_θ which is expressed by

(35) $$P(x_\theta) = (2\pi\sigma_\theta)^{-\frac{1}{2}}\, e^{-x_\theta^2/(2\sigma_\theta^2)}$$

where σ_θ = the variation of x_θ. Usually x_θ is assumed to have the properties of stationarity and ergodicity. Using the above, one can obtain (see Appendix):

$$\text{(36)} \qquad \left.\frac{S}{N}\right]_I = \frac{\eta^4 \psi_s^2 \, (1 - \sigma_\theta^2 \omega_o^2)}{\frac{\omega_L}{2\Delta\omega_{IF}} F(\psi_s, \psi_{nj}) + \sigma_\theta^2 \eta^4 \psi_s^2 \cdot \frac{\omega_L \Delta\omega_{IF}}{\psi}}$$

for the output SNR. Here $F(\psi_s, \psi_{nj})$ stands for the denominator of the expression in Eq. (9).

From the above it can be seen that if σ_θ is large, considerable degradation of the SNR will result from the increase in the noise level, and the simultaneous decrease in the signal level. If $\sigma_\theta^2 \omega_o^2$ approaches unity, the SNR would fall to a very small value. Thus, for good SNR, σ_θ should be much less than $1/\omega_o$.

III. COMPARISON WITH THE DICKE RADIOMETER

Some of the characteristics of the correlation radiometer have been compared with those of the Dicke radiometer in Ref. 1. Here an individual comparison of the IF type and the ENV type of system with the Dicke system will be made.

In the case where the input signals to both channels of the correlation radiometer are small as compared to the noise signals in each channel, the minimum detectable temperature of the IF type of correlation radiometer can be shown to be (when the gain fluctuations are included):

$$\text{(37)} \qquad T_{min}\Big]_{fI} = \kappa_I \left[\frac{T_n}{\sqrt{\alpha}} \left(1 + \frac{1}{2} \sum_{j=1}^{2} \frac{\psi A j}{A_{oj}^2} \right) \right]$$

where κ_I is a constant. (In the following, the subscript will denote the Dicke radiometer, I the IF type, and E the envelope type of correlation radiometer.) The sensitivity of the Dicke radiometer is given by

$$\text{(38)} \qquad T_{min}\Big]_{fD} = \kappa_D \left[\frac{T_n}{\sqrt{\alpha}} + \frac{\Delta G}{G_o} (\Delta T_A + T_S) \right]$$

where κ_D is another constant of the same order of magnitude as κ_I. T_n is the equivalent system noise temperature, ΔG is the gain fluctuation factor, G_o is the mean value of the gain, ΔT_A is the temperature difference between the source and the load temperatures, and T_s is the noise temperature of the source. It can be seen that the first terms in Eqs. (37) and (38) can be regarded as being similar terms. The second terms are also comparable, since the term

$$\frac{1}{2} \sum_{j=1}^{2} \left(\frac{\psi Aj}{A_{oj}^2} \right)$$

may correspond to $\Delta G/G_o$, and the terms T_n/α and $(\Delta T_A + T_s)$ can be more or less considered to be equivalent. As was shown in the preceeding sections, the ENV type of radiometer had a poorer sensitivity than the IF type of radiometer by a factor of $(2)^{-\frac{1}{4}} \cdot (\alpha)^{-\frac{1}{2}} \cdot (\beta)^{-\frac{1}{4}}$.

In the case where one or the other of the signal inputs is no longer small, e.g., $R_1 \gg 1 \gg \eta^2 R_2$, it can be shown that the minimum detectable temperatures, for the case where output SNR >> 1, is:

$$\text{(39)} \qquad \Delta T_I = \frac{T_n}{4\alpha\eta^2} \text{ (IF type)} \qquad (T_n = F_e\, T_o)$$

$$\text{(40)} \qquad \Delta T_E = \frac{T_n^2}{8\beta\eta^4} \text{ (ENV type)} \qquad (T_n = F_e\, T_o)$$

which shows that the sensitivity depends upon T_n, and η. If we would compare the sensitivity of the above systems with a Dicke system looking at the larger source, we find that for the Dicke system;

$$\text{(41)} \qquad \Delta T_D = \kappa_D \cdot T_1/\alpha .$$

where F_e is the equivalent noise figure of the receivers in all cases, and T_o is the standard temperature (commonly taken to be 290 degrees K).

The latter relation can be derived from the equation for the output SNR in Goldstein's paper.[2] Comparing the above, it is seen that the correlation radiometer would have a lower sensitivity than the Dicke radiometer because $(\eta^2 T_1/T_n) < 1$.

For the case where the output SNR is close to unity, T_{min} is given by:

(42) $$T_{min}\Big]_I = \frac{F_e T_o}{2\alpha} \quad \text{(IF type)}$$

(43) $$T_{min}\Big]_E = \frac{F_e T_o}{\sqrt{2\beta}} \quad \text{(ENV type)} \quad .$$

Since the sensitivity in the Dicke case is expressed by

(44) $$T_{min}\Big]_D = \kappa_D \frac{F_e T_o}{\sqrt{\alpha}} \quad ,$$

we see that the IF type of radiometer would be more sensitive than the Dicke radiometer in this case by a factor of $\sqrt{\alpha}$, while the ENV type of radiometer would have a sensitivity on the same order of magnitude as the Dicke radiometer, except for factor of order unity due to constant term.

IV. CONCLUSIONS

The two basic types of correlation radiometer, the IF type and the envelope-detection type (ENV type), have been discussed and compared. The signal-to-noise ratio at the output and the minimum detectable temperature increments have been expressed in terms of the signal-to-noise ratios at the inputs. These results have shown that the IF type of radiometer is superior to the ENV type of system in terms of the sensitivity and should be used except where the phase information of the two input signals is uncorrelated. The effects of gain fluctuations upon the minimum detectable temperature have been compared for the IF, the ENV, and the Dicke types of systems. It was found that the IF and the Dicke systems gave comparable results but that the effects of gain fluctuations in the ENV type of system was worse than in the IF type of system. It was also shown that phase fluctuations in the receivers could result in large degradations in the system sensitivity.

Correlation technique[3, 4, 5, 6] and its application to radiometers have been discussed in previous work.[2, 7, 8] Here the emphasis is on determining the most useful applications of the correlation-type radiometers. These would be in interferometer systems (IF type) in the millimeter or submillimeter wavelength regions where the elimination of the microwave switch which is used in the Dicke system is the main advantage which the correlation radiometer can claim; and for the studying of the correlation (ENV type of system) between the signals received from two different sources such as the sun and the moon.

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APPENDIX - Evaluation of the effects of the receiver phase fluctuations

Starting with the expressions for the inputs to the correlator:

(A-1) $$U_1(t) = s(t) + n_1(t)$$

(A-2) $$U_2(t) = \eta s(t+\theta + x_\theta) + n_2(t+\theta+x_\theta),$$

the correlation function of the correlator output is found to be

(A-3) $$\phi_m = \int [\eta^2 \{\phi_s^2(\theta+x_\theta) + \phi_s{}^2(\tau) + \phi_s(\theta+\tau+x_\theta)\phi_s(\theta+x_\theta-\tau)\} + \phi_s(\tau)\phi_{n2}(\tau) + \eta^2\phi_s(\tau)\phi_{n1}(\tau) + \phi_{n1}(\tau)\phi_{n2}(\tau)]P(x_\theta)dx_\theta .$$

Expanding the above in Taylor Series about $\theta = \theta_r$, the following are obtained,

(A-4) $$\phi_m = \phi_{mo} + \sigma_\theta^2 \Delta\phi_m ,$$

where $\Delta\phi_m$ and ϕ_{mo} respectively represent the correlation functions with and without the phase fluctuation. $\Delta\phi_m$ is given by

(A-5) $$\Delta\phi_m = \eta^2\left[\phi_s'^2(\theta) + \phi_s(\theta)\phi_s''(\theta) + \frac{1}{2}\{\phi_s''(\tau+\theta)\phi_s(\theta-\tau) + 2\phi_s'(\theta+\tau)\phi_s'(\theta-\tau) + \phi_s(\theta+\tau)\phi_s''(\theta-\tau)\}\right]$$

where prime means the derivatives with respect to τ. Using the same form of the autocorrelation functions of signal and noise as Eq. (6), and after removing dc-components, we obtain:

(A-6) $$\Delta\phi_m\Big]_{dc} = \eta^2\psi_s^2\, e^{-\omega_s(|\tau+\theta| - |\tau-\theta|)}\,\omega_s^2 \cos 2\omega_o\theta .$$

After some manipulations similar to those used for obtaining Eq. (8), we find the following terms at the output of correlator;

(A-7) dc-term: $\eta^2 \psi_s^2 \, e^{-\Delta\omega_{IF}\theta} [\cos^2\omega_o\theta - \sigma_\theta^2 \omega_o^2 \cos 2\omega_o\theta]$

(A-8) ac-term: $$F(\psi_{sj}\,\psi_{nj}) \frac{\omega_L}{2\Delta\omega_{IF}} + \sigma_\theta^2 \eta^2 \psi_s^2 \frac{\omega_L \Delta\omega_{IF}}{4} \left[\frac{\Delta\omega_{IF}}{\omega_L}(1-e^{-2\omega_L\theta}) + 1\right] \cdot e^{-\Delta\omega_{IF}\theta} \cos 2\omega_o\theta \, .$$

Finally, for the SNR at the correlator output, we have, for $\theta = 0$,

(A-9) $$\left.\frac{S}{N}\right]_I = \frac{\eta^2 \psi_s^2 (1 - \sigma_\theta^2 \omega_o^2)}{F(\psi_s, \psi_{nj}) \frac{\omega_L}{2\Delta\omega_{IF}} + \sigma_\theta^2 \eta^2 \psi_s^2 \frac{\omega_L \Delta\omega_{IF}}{4}} \, .$$

(In the above calculations it has been assumed that $\omega_s = \omega_{N_1} = \omega_{N_2} = \Delta\omega_{IF}/2 << \omega_o$ and the terms above the second order have been omitted.)